Annihilation of Positrons in Rare Gases — A Phenomenological Study

P. S. Grover

Department of Physics, University of Delhi, India

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The life time, λ, of positrons in rare gases under the simultaneous influence of a steady magnetic and an alternating electric field is studied. λ depends on the frequency and attains a maximum when the frequency equals the cyclotron frequency. The extent of this dependence is quite sensitive to the positron atom momentum transfer rates.

The annihilation of positrons in rare gases under the influence of temperature, electric fields and magnetic fields has been studied by several workers [1 – 16], as such studies provide tests of models of positron-atom interaction. Here, we propose a method in which a steady magnetic and on alternating electric field are applied to an atomic gas in which positrons undergo scattering and annihilation processes. The energy of the positrons is assumed to be smaller than the positronium formation threshold, so that they suffer only elastic collisions and eventually annihilate with the electrons of the atoms. In the presence of a uniform alternating electric field, $\mathbf{E} = \mathbf{E}_0 \exp(i\,\omega\,t)$, and a magnetic field, $\mathbf{B}$, the behaviour of the positrons is described by the Boltzmann equation [11, 17]:

$$\frac{\partial f(\mathbf{V},t)}{\partial t} + \frac{e}{m}\left(\mathbf{E} + \frac{\mathbf{V}\times\mathbf{B}}{c}\right)\cdot\frac{\partial f(\mathbf{V},t)}{\partial \mathbf{V}} = \left(\frac{\delta f}{\delta t}\right)_m + \left(\frac{\delta f}{\delta t}\right)_a . \qquad (1)$$

Here e, m and $v = |\mathbf{V}|$ are the positron charge, mass and velocity, respectively, and c is the velocity of light. $f(\mathbf{V},t)\,\mathrm{d}\mathbf{V}\,\mathrm{d}t$ specifies the number of positrons at time t with velocity $\mathbf{V}$ in the ranges $\mathrm{d}t$ and $\mathrm{d}\mathbf{V}$. $\left(\frac{\delta f}{\delta t}\right)_\mathrm{m}$ and $\left(\frac{\delta f}{\delta t}\right)_\mathrm{a}$ are the scattering and annihilation terms, respectively.

To solve (1), we write $f(\mathbf{V},t) = f_0(v,t) + \mathbf{V}\cdot\mathbf{f}_1 \exp(i\,\omega\,t)$ [17], where $\mathbf{f}_1$ is a function of v and $|\mathbf{f}_1| \ll f_0$. Using this expansion and evaluating the scattering and annihilation terms following ref. [17], (1) can be put in the form

$$\frac{\partial f_0(v,t)}{\partial t} = \frac{1}{v^2}\frac{\partial}{\partial v} \cdot \left[\left\{\frac{v^2 a_\mathrm{eff}^2}{3\,\nu_\mathrm{m}} + \frac{k\,T}{M} v^2\,\nu_\mathrm{m}(v)\right\}\frac{\partial f_0}{\partial v} + \mu\,\nu_\mathrm{m}\,v^3 f_0\right] - \nu_\mathrm{a} f_0 , \qquad (2)$$

where

$$\nu_\mathrm{m}(v) = n\,v\,\sigma_\mathrm{m}(v), \quad \nu_\mathrm{a}(v) = n\,v\,\sigma_\mathrm{a}(v),$$

and

$$\mu = m/M .$$

$\sigma_\mathrm{m}(v)$ and $\sigma_\mathrm{a}(v)$ are the momentum transfer and annihilation cross-sections, respectively, M the atomic mass and n the number density. a_eff^2 is the effective root-mean-square positron acceleration given by

$$a_\mathrm{eff}^2(\omega) = \frac{1}{2}\left[a_\mathrm{p}^2\left\{1 + \frac{\omega^2}{\nu_\mathrm{m}^2}\right\}^{-1} + \frac{a_\mathrm{t}^2}{2} \cdot \left\{\left[1 + \frac{(\omega-\omega_\mathrm{c})^2}{\nu_\mathrm{m}^2}\right]^{-1} + \left[1 + \frac{(\omega+\omega_\mathrm{c})^2}{\nu_\mathrm{m}^2}\right]^{-1}\right\}\right], \qquad (3)$$

where $\mathbf{a}_\mathrm{p} = e\,\mathbf{E}_{0\mathrm{p}}/m$ and $\mathbf{a}_\mathrm{t} = e\,\mathbf{E}_{0\mathrm{t}}/m$. $\mathbf{E}_{0\mathrm{p}}$ and $\mathbf{E}_{0\mathrm{t}}$ are the components of the electric field parallel and transverse to the direction of the magnetic field, $\omega_\mathrm{c} = e\,B/m\,c$. When $\omega = 0$,

$$a_\mathrm{eff}^2(0) = \left[a_\mathrm{p}^2 + \frac{a_\mathrm{t}^2}{1 + \omega_\mathrm{c}^2/\nu_\mathrm{m}^2}\right],$$

and if in addition $a_\mathrm{p} = 0$, (2) reduces to (3) of Ref. [15].

After a lapse of sufficient time (greater than the slowing down time of positrons), we can write $\partial f_0/\partial t = -\lambda\,F(v)$ [2]. Using this in (2), and after an integration:

$$\left[\frac{a_\mathrm{eff}^2(\omega)}{3\,\nu_\mathrm{m}^2} + \frac{k\,T}{M}\right]\frac{\partial F}{\partial v} + \mu\,v\,F(v) = \frac{1}{\nu_\mathrm{m}\,v^2}\int_0^v [\nu_\mathrm{a}(v') - \lambda]\,v'^2\,F(v')\,\mathrm{d}v' . \qquad (4)$$

This equation can be solved to find $F(v)$, and then λ is obtained from the relation

$$\lambda = \left[\int_0^\infty \nu_\mathrm{a}(v)\,v^2\,F(v)\,\mathrm{d}v\right]\left[\int_0^\infty v^2\,F(v)\,\mathrm{d}v\right]^{-1} . \qquad (5)$$

λ is the annihilation decay constant and λ^{-1} the life time of the positrons. To solve (4), we assume that in the lowest approximation $\lambda \approx \nu_\mathrm{a}$, so that the integral on the right hand side of (4) can be ignored and the solution of the resultant equation becomes

Reprint requests to Dr. P. S. Grover, Department of Physics, University of Delhi, Delhi-110007, Indien.

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$$F(v) = \text{const} \exp \left[-\mu \int_0^v \mathrm{d}v\, v \left\{ \frac{a_{\text{eff}}^2}{3\,\nu_{\text{m}}^2} + \frac{k\,T}{M} \right\}^{-1} \right] . \tag{6}$$

Thus, λ can be obtained from (5). This procedure leads to reasonable values of λ [9, 10, 12].

We apply this method to find λ and its dependence on ω at constant B. As at present we are interested in a first insight, we take $\nu_{\text{m}} = b_1$ (a constant) and $\nu_{\text{a}} = b_2/\sqrt{v}$, where b_2 is also a constant [1, 9]. With this, (5) can be integrated, using (6) for $F(v)$, to yield

$$\lambda(\omega) = 1.02\, b_2 \left[\frac{\mu}{2} \left\{ \frac{a_{\text{eff}}^2}{3\,b_1^2} + \frac{k\,T}{M} \right\}^{-1} \right]^{1/4} . \tag{7}$$

We consider the case $T = 300$ K and $n = 1$ amagat and calculate $\gamma = \lambda(\omega)/\lambda(0)$, where $\lambda(0)$ relates to constant crossed electric and magnetic fields. Values of this ratio have been obtained for several frequencies of the electric field in the range 10^{10} to 2×10^{12} radians s^{-1}. The field strengths considered are $E_{0\text{p}} = 0$, $E_{0\text{t}} = 30$ V/cm/amagat, and $B = 1.15$ T. Cyclotron frequency $\omega_{\text{c}} = 2 \times 10^{11}$ radians s^{-1}. The values taken for the momentum transfer rates are 2×10^{10}, 5×10^{10} and $10^{11}\ \text{s}^{-1}$.

The dependence of the ratio γ on frequency is presented in Figure 1. It is obvious that λ depends quite sensitively on the frequency of the electric field. As ω nears ω_{c} from either side, λ decreases and hence the life time increases. This is because the effective field ($= m\, a_{\text{eff}}/e$) 'seen' by the positrons enhances with frequency and becomes quite large at resonance. The decrease of λ is more rapid for lower momentum transfer rates. This is because the energy transfer from the field to the positrons at resonance ($\omega = \omega_{\text{c}}$) is much greater at lower ν_{m}. The relative changes in λ defined as: $\Delta_1 = \gamma(10^{10}) - \gamma(2 \times 10^{11})$, $\Delta_2 = \gamma(2 \times 10^{12}) - \gamma(2 \times 10^{11})$ are:

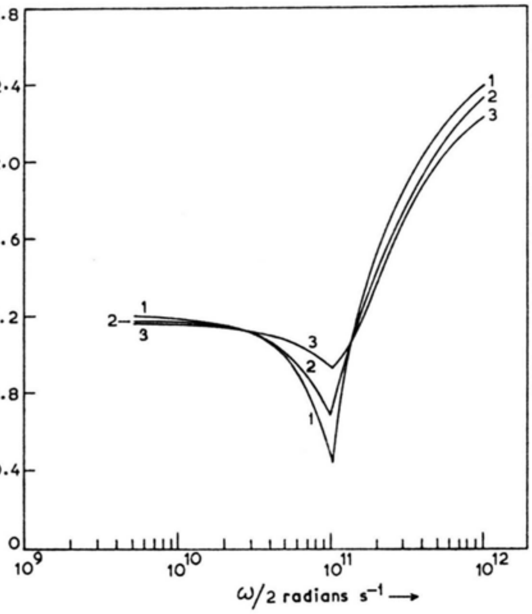

Fig. 1. Dependence of ratio $\gamma = \lambda(\omega)/\lambda(0)$ on the frequency of the electric field. Curves 1, 2, 3 are for $\nu_{\text{m}} = 2 \times 10^{10}$, 5×10^{10} and $10^{11}\ \text{s}^{-1}$, respectively.

$\Delta_1 = 0.743$, 0.481, 0.244 and $\Delta_2 = 1.940$, 1.642, 1.312 for $\nu_{\text{m}} = 2 \times 10^{10}$, 5×10^{10} and $10^{11}\ \text{s}^{-1}$, respectively. Thus, the effect of frequency is expected to be most pronounced in gases with low momentum transfer rates, like helium.

It may be interesting to look into the present technique experimentally for the study of positron-gas systems. Such studies for electron-gas systems have been quite useful [18].

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